고정 중간점을 허용하는 프랙탈 기법에 대한 연구*

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요 약

2차원 및 3차원 그래픽스 분야에서 지형 형상화를 위하여 프랙탈 기법이 많이 사용되어 왔다. 프랙탈 기법에서는 랜덤효과를 이용하여 가상적인 지형을 형상화하는 것이 일반적인 방법으로, 3차원 특정지형, 예를 들어서 제주도나 울릉도와 같은 실제지형을 형상화하기는 힘들다. 그러나 특정지형에 대한 최소한의 3차원 데이타를 제어점으로 사용하여 이를 프랙탈 기법에 적용함으로써 특정지형과 유사한 3차원 지형을 만들어 낼 수가 있다. 여기서는 3차원에서의 고정 중간점에 대한 지정과 이를 프랙탈 알고리즘에서 어떻게 구현하는지에 대한 내용을 보인다.

A Revised Fractal Technique With Fixed Midpoints For A Specific Terrain Model

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ABSTRACT

In 2D and 3D computer graphics, fractal techniques have been applied to terrain models. In general, a specific 3D terrain model such as Cheju or Uleung Island could not be formulated by statistical fractals owing to the random effects. However, by locating the control points on the edges and the surface of a specific terrain such as Cheju Island, a similar shape of the terrain model can be simulated. This paper presents the way of formulating a specific 3D terrain model by the statistical fractals with fixed midpoints.

1. INTRODUCTION

Generating 3D objects or structures is an important area in computer graphics. One way of representing 3D objects is to use 3D polygon mesh model of an object. To dercribe an object, a large number of data should be prepared to get the polygon mesh model. For example, the Utah teapot, a familiar object that has become a kind of benchmark in computer graphics, has the data consisting of 306 world coordinate vertices[6]. In this case, the more data we have, the more realistic object we can obtain.

A specific terrain model such as Cheju Island could be formulated by preparing data of the 3D world coordinate vertices too. This technique assumes that objects are collections of polygons whose surfaces are described by linear functions or higher order polynomials such that this method has established itself as a convenient and powerful technique for modeling smooth and man-made specific shape such as car, building etc. However, preparing the 3D world coordinate vertices of 3D objects are generally very tedious job and how many data should be prepared is dependent upon the level of detail.

In general, many objects, such as terrain,

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mountains and clouds, have irregular or fragmented features[3]. Such objects have been modeled using the fractal geometry methods developed by Mandelbrot [3]. A fractal geometric curve contains an infinite variety of detail at each point along the curve. Mandelbrot used a term fractal geometry which was used to describe the attributes of certain natural phenomena like coastlines. For example, trees in a forest landscape can be sensibly generated procedurally.

In 3D computer graphics, fractal techniques have been applied to terrain models. Fractal models have become popular for recreating a wide variety of the shapes found in nature[5]. Most fractal models feature a stochastic component, making them well suited to generating nonsmooth, irregular shapes, such as mountainous terrain[2].

Most fractal terrain models have been based on one of five approaches: midpoint displacement, Poisson faulting, Fourier filtering, successive random additions, and summing band-limited noises[7]. The approach presented here is of the first type, in which the midpoints are selected by deterministic manner.

2. RELATED WORK

Midpoint displacement algorithms or what we call, subdivision algorithms, are based on a formulation by Fournier et al. [1,2] that recursively subdivides a single line segment. These methods are standard in fractal geometry. Musgrave, Kolb, and Mace[7] classify the various midpoint displacement techniques by locality of reference: wireframe midpoint displacement, tile midpoint displacement, generalized stochastic subdivision, and unnested subdivision.

Wireframe subdivision is used in triangle subdivision scheme and involves the interpolation between two points in the subdivision process while tile midpoint displacement involves the interpolation of 3 or more non-collinear points. The above two methods are generally efficient and easy to implement, however have fixed lacunarity and are nonstationary due to nesting.

Generalized stochastic subdivision interpolates several local points, constrained by an autocorrelation function[8]. Instead of displacing each midpoint independently, Lewis adds correlated Gaussian noise. This noise alleviates the artifacts due to spatially nonstationary statistics across triangles that are sometimes evident with the other methods. Miller proposed an unnested subdivision wherein unnested means that successive levels of subdivision retain no points from previous levels[9]. However, this is not strictly a midpoint subdivision scheme. The former is flexible but very hard to implement, while the latter features fixed lacunarity and is simple to implement.

Musgrave[7] presented "noise technique"which features locally independent control of the frequencies composing the surface, and thus local control of fractal dimension and other statistical characteristics. This method is intermediate in difficulty of implementation. Szeliski and Terzopoulos[10] developed "constrained fractals", a hybrid of splines and fractals which intimately combines their complementary features. Their method combines deterministic splines and stochastic fractals to inherit complementary features. The constrained fractals avoid the artifacts that are introduced during the subdivision process, and they can assimilate constraints at any resolution.

The common approach mentioned above for obtaining some control is to first triangulate a given set of points, then add fractal texture by recursively subdividing and randomly perturbing the subtriangles. But this method produces annoying visual artifacts because the spatial statistics are non stationary across the original triangle boundaries. Therefore, the above methods are very suitable for formulating free-form shape but not applicable to a specific object [10]. This paper, therefore, proposes a revised subdivision fractal technique with fixed midpoints which simultaneously provides both control and detail. The midpoints prepared are usually located on the edge or surface on a specific object. With this method, a realistic and visually satisfactory shape of a specific terrain model could be generated from a very small database.

3. THE ALGORITHM WITH FIXED MIDPOINTS

In statistical fractals, a recursive subdivision procedure is applied to each facet, to a required depth or level of detail, and a convincing terrain model results. Subdivision in this context means taking the midpoint along the edge between two vertices and perturbing it along a line normal to the edge [6]. The result of this is to subdivide the original facets into a large number of smaller facets.

To extend this procedure to triangles in 3D space, each edge is treated to generate a displacement along a midpoint vector that is normal to the plane of the original facet. And each facet has a random orientation in 3D space about the original facet orientation. Using this technique we can take a smooth pyramid, say, made of large triangle faces and turn it into a rugged mountain or a free-form terrain surface.

The recursive subdivision algorithm applied to triangles in 3D space need 3 basic vertex points. These points are not altered as the procedure is performed. Mid-points which are generated based on the edges cannot be anticipated because of their randomness. A midpoint is generated by repeatedly applying a specified subdivision algorithm to points within a region of a space.

Although the recursive subdivision algorithm can proceed infinitely, a terrain surface is usually generated with a finite number of iterations. This number of iteration can be a index of the level of detail of an object. The amount of detail included in the final display of the curve usually depends on the number of iterations performed and the resolution of the display system. Therefore, the number of triangles and midpoints are increased as the iteration proceeds. The equations of counting the number of triangles to draw and the total number of points to compute are as follows.

$$N_t = \sum_{k=1}^{n} K \quad n = 2^i \qquad (1)$$

$$N_p = 0 \qquad \text{if} \quad i = 0 \qquad (2)$$

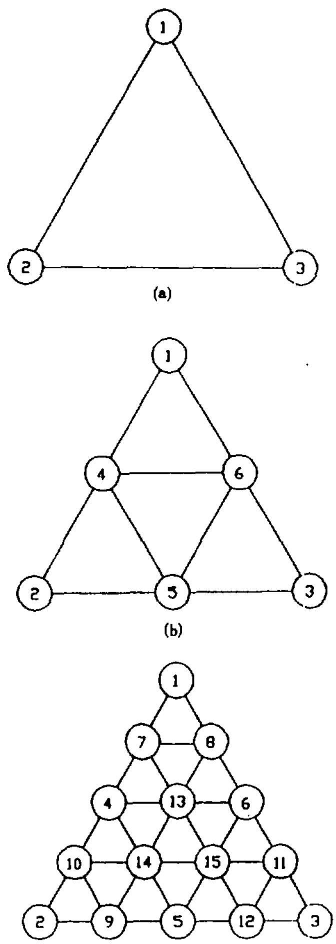
$$= \sum_{k=1}^{n} K \quad \text{otherwise,} \ n = (2^i + 1)$$

where i is the number of iterations, Nt and Np are the number of triangles and the number of points. Equation (1) shows the minimum number of triangles to draw the fractal surface. However, in order to apply illumination model of determining the lighting and color for each face to fractal surface, the number of triangles to draw is as follows in equation (3).

$$N_t = \sum_{k=1}^{n} K + \sum_{k=1}^{n-1} K \qquad n = 2^i \qquad (3)$$

As shown in equation (2), the total number of points to draw a terrain model based on triangles in 3D space increases exponentially like 3, 6, 15, 45, 153,, as the number of iterations increases. The number of midpoints obtained by the subdivision algorithm are 3, 9, 20, 108,, depending on the number of iterations. If we select some points as midpoints to reflect the characteristics of an object, we could alleviate the random effects such that a similiar object could be formulated.

The midpoint displacement algorithm starts out with a base triangle and heights chosen at the three vertices as shown in (Figure 1-a). The triangle is subdivided into four subtriangles at the first iteration and three new points are made as shown in (Figure 1-b). At the second iteration, 9 new points are made and also shown in (Figure 1-c). As shown in (Figure 1), the sequence of the midpoints is very crucial to the subdivision algorithm.



(Figure 1) Midpoints

By supplying the midpoins in series as shown in (Figure 1) with the X, Y, and Z value, a specific object or terrain simulation is performed. From third iteration, the midpoints are made by the subdivision algorithm without control points, therefore, we could get lots of geometric data describing the object or terrain. Even though the random effects are permitted at this time, the overall structure of a specific shape can be maintained.

The revised subdivision algorithm with fixed midpoints are shown in (Figure 2). Even though there are several ways to compute the

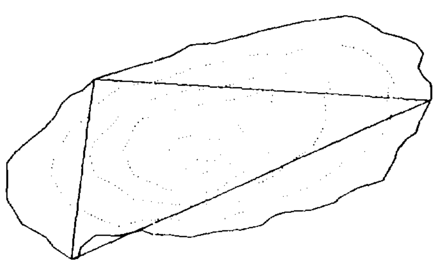
```
double ax[15],ay[15],az[15];
int     Max_iter;
int     stack_count=0;
const resolution;
procedure fractal(x, y, z, roughness,iteration:re-al) ;
begin
  if(length of line segment< resolution) then ite-ration
  =0 ; if(iteration = 0)
    begin
      Move to first point;
      Line to second point;
      Line to third point;
      Line to first point;
    end

  else
    begin
      {switch(Max iter n) {
        case 0:insert 4th,5th,6th midpoints; break;
        case 1:
        {stack_count=stack_count+1 ;
             if(stack_count= 1)insert 7th, 8th,
             13th midpoints;
             if(stack_count= 2)insert 9th, 10th,
             14th midpoints;
             if(stack_count= 3)insert 11th, 12th,
             15th midpoints;
             if(stack_count= 4)insert 13th, 14th,
             15th midpoints;}
             default:compute roughness and
                   midpoints;}
      fractal(x,y,z,roughness,iteration-1);
      fractal(x,y,z,roughness,iteration-1);
      fractal(x,y,z,roughness,iteration-1);
      fractal(x,y,z,roughness,iteration-1);
    end
  end;
```

(Figure 2) A revised algorithm with fixed midpoints.

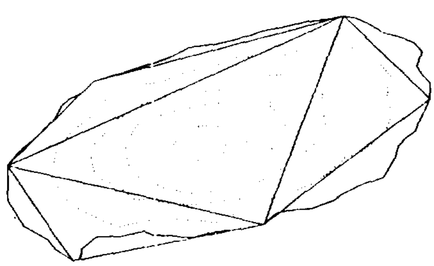
height of the terrain surface, the height of the terrain surface is provided in this algorithm by simply interpolating from the heights of its two neighbor points and then placed in the usual fashion. The direction of the displacement of height is set normal to the X-Y plane. Note that the concept of counting stack level is introduced to assign the midpoints values to new points because of recursive call in (Figure 2).

4. APPLICATIONS-CHEJU ISLAND

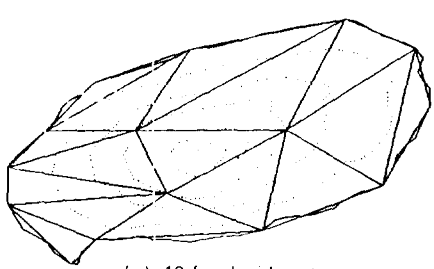
Based on the algorithm in (Figure 2), the terrain model of Cheju Island is simulated as an example. Cheju Island is the largest Island in Korea and located in the south sea of the Korea Peninsula. The surface of Cheju Island is relatively smooth and the Mt. Hanla is located in the center of Island.



(a) 3 vertex points



(b) 3 fixed midpoints



(c) 12 fixed midpoints

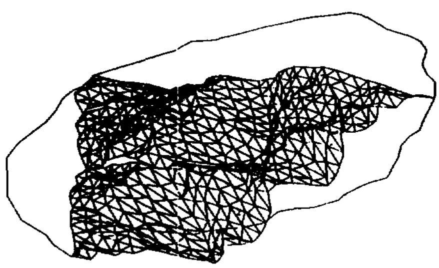
(Figure 3) Cheju Island with Fixed Midpoints

To depict Cheju Island by the statitical fractals, 3 vertex points are selected as shown in (Figure 3-a). Based on these points, imaginary 3D geometry data of Cheju Island are made by the recursive subdivision procedure. (Figure 4) shows the terrain surface generated by the algorithm with w=0.5 and N(i)=5 where w is the value of weighting factor and N(i) is the number of iterations.

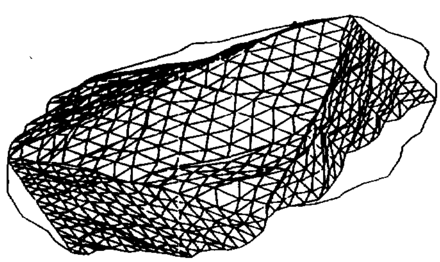
Out line in (Figure 4) indicates the shape of Cheju Island drawn by the geometric data obtained from Cheju Island contour map. Given three vertex points, a fractal surface is generated for the area between the vertex points. (Figure 4) shows that the shape of Cheju Island drawn by geometric data and the shape of Cheju Island drawn by the fractals are quite different. Differences in the appearences of these objects are due to the kind of random effects used to provide object irregular features. To reduce the amount of differences, 3 midpoints of the line segments are forced to locate on the outline of Cheju Island as shown in (Figure 3-b). In (Figure 3-c), 12 midpoints are selected to draw Cheju Island.

(Figure 5) shows the shape of Cheju Island with 3 vertex and 3 midpoints wherein w=0.3 and N(i)=5 are used. The value of weighting factor is reduced by 0.2 compared to that of (Figure 4.) This is because the polygon made by 3 vertex and 3 midpoints are closer to the coastline of Cheju Island than the triangle made by three vertex points case. The closer to coastline, the less rugged line segment is needed.

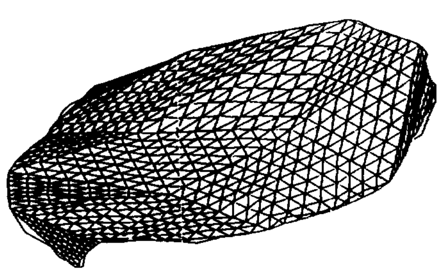
(Figure 6) shows the top view of Cheju Island generated with 3 vertex and 12 midpoints. The weighting factor is −0.1 and the number of iterations is 5. In this case, differences in the shape of two objects are sharply reduced because of the added midpoints. Even though the Z values of points in the terrain are interpolated from the Z values of its two neighboring points, the randomness of Z values are also given by the weighting factor. Therefore, we could get the similar shape of Cheju Island modeled with 1024 faces made by 3 base vertex points and 12 fixed midpoints by the algorithm. When taking a closer view of the terrain, the surface no longer appears flat. (Figure 7) depicts the front view of Cheju Island shown in (Figure 6).



(Figure 4) Surface generation with 3 vertex points.



(Figure 5) Surface generation with 3 midpoints



(Figure 6) Surface generation with 12 midpoints

(Figure 7) The Front View of Cheju Island

5. SUMMARY

Many techniques are used to draw an object on a computer. Some objects must be drawn in detail, but the others are not necessarily. In this paper, a revised fractal method with fixed midpoints is presented. By locating the control points on the edges of an object, we could get not exactly the same but the similar shape of an object. As a case study, Cheju Island has been considered in the paper. Cheju Island has smooth coastline and the relatively flat terrain surface. Therefore, the shapes similar to the Cheju Island could be made by assigning a small weighting factor.

Even though we could get the similar shape of an object by the above method, there are several drawbacks. First, how to select the control points on an object and how to choose the value of weighting factor are uncertain. Secondly, because of randomness, only the similar shape of an object can be obtain. Finally, the geometric data generated by the above method is an imaginary data rather than real data such that we cannot use the data in GIS.

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